



Српско научно математичко друштво

Kongres mladih matematičara u Novom Sadu

03 – 05. oktobar 2019.
Novi Sad, Srbija

Knjiga sažetaka

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Plan rada

Četvrtak, 03. oktobar 2019.

Srpska akademija nauka i umetnosti, Ogranak u Novom Sadu, Nikole Pašića 6, Novi Sad

14:00-15:00		Registracija učesnika
15:00-15:15		Otvaranje Kongresa
15:15-16:00	Dušan Jakovetić	<i>Distributed optimization and learning</i>
16:00-16:45	Luka Milićević	<i>Multilinear maps on multilinear varieties</i>
16:45-17:00		pauza za kafu
17:00-17:45	Marko Petković	<i>Iterative methods and neural networks for the computation of generalized inverses</i>
17:45-18:30	Bojan Prangoski	<i>Quasi-analytic representation theory of $(\mathbb{R}^d; +)$ over quasi-complete locally convex spaces</i>
19:30		svečana večera u restoranu "Fontana"

Petak, 04. oktobar 2019.

Departman za matematiku i informatiku, Trg Dositeja Obradovića 4, Novi Sad

08:30-09:00 Registracija učesnika

Amfiteatar I, predsedavajući: Bojan Prangoski

09:00-09:20	Suzana Aleksić	<i>Frames for Hilbert and Banach spaces</i>
09:20-09:40	Sanja Atanasova	<i>Characterization of wave front sets via Stockwell transform</i>
09:40-10:00	Pavel Dimovski	<i>Modulation spaces related to translation-invariant Banach spaces of quasi-analytic ultradistributions</i>
10:00-10:20	Lenny Neyt	<i>Characterizing the nuclearity of Gelfand-Shilov spaces</i>
10:20-10:40	Zorica Milovanović Jeknić	<i>Nonlocal boundary value problem</i>

Amfiteatar VII, predsedavajući: Marko Petković

09:00-09:20	Marija Krstić	<i>Stability of the Stochastically Perturbed Tumor-Immune Interaction Model with Delay</i>
09:20-09:40	Jasmina Đorđević	<i>A stochastic analysis of the impact of fluctuations in the environment on pre-exposure prophylaxis for HIV infection</i>
09:40-10:00	Marija Milošević	<i>Comparison of the Euler-Maruyama and backward Euler methods for a class of pantograph stochastic differential equations</i>
10:00-10:20	Dušan Đorđević	<i>L^p and almost sure convergence of an approximate method for stochastic differential equations</i>
10:20-10:40	Vuk Vujović	<i>Stohastički Heroinski model</i>

10:40-11:00 pauza za kafu

Amfiteatar I, predsedavajući: Bojan Bašić

11:00-11:20	Lazar Milenković	<i>Aproksimacioni algoritmi za minimizaciju uskog grla u asimetričnoj verziji problema trgovačkog putnika</i>
11:20-11:40	Anna Slivková	<i>Hešov broj je neograničen za $d \rightarrow \infty$</i>
11:40-12:00	Kristina Ago Balog	<i>O jako palindromičnim rečima: ternarni slučaj</i>
12:00-12:20	Stefan Hačko	<i>O nekim aritmetički interesantnim kolekcijama permutacija</i>
12:20-12:40	Danijela Mitrović	<i>Emulaciona ekvivalencija kombinatornih igara</i>

Amfiteatar I, predsedavajući: Filip Tomić

16:00-16:20	Milica Žigić	<i>Wick-type nonlinearities in stochastic evolution equations with randomness</i>
16:20-16:40	Katarina S. Kostadinov	<i>Existence and asymptotic behavior of q-regularly varying solutions of nonlinear second order q-difference Thomas-Fermi equation</i>
16:40-17:00	Valentina Timotić	<i>Logarithmic (translationally) rapidly varying sequences and selection principles</i>
17:00-17:20	Nevena Petrović	<i>Anti-Gaussian quadrature rule for trigonometric polynomials</i>
17:20-17:40	Tatjana V. Tomović	<i>Multiple Orthogonality and Applications in Numerical Integration</i>

Amfiteatar VII, predsedavajući: Ivana Đurđev

16:00-16:20	Kristina Asimi	<i>Obećanja svode konačne probleme na beskonačne</i>
16:20-16:40	Vladica Andrejić	<i>Algoritmi za ostatke levog faktorijela i Kurepina hipoteza</i>
16:40-17:00	Miloš Milovanović	<i>Intuicionističko zasnivanje matematike i primene u muzici, arhitekturi, obrazovanju. . .</i>
17:00-17:20	Simona Kašterović	<i>Kripkeove semantike za lambda račun sa parovima i sumama</i>
17:20-17:40	Nenad Stojanović	<i>Metric logics</i>

20h druženje u klubu "Giardino", Bulevar Mihajla Pupina 1, poslednji sprat

Subota, 05. oktobar 2019.

Departman za matematiku i informatiku, Trg Dositeja Obradovića 4, Novi Sad

Amfiteatar I, predsedavajući: Kristina Ago Balog

09:00-09:20	Boriša Kuzeljević	<i>Globalna teorija ultrafiltera</i>
09:20-09:40	Samir Zahirović	<i>O obogaćenom stepenom grafu grupe</i>
09:40-10:00	Emir Zogić	<i>Laplacian resolvent energy of graphs</i>
10:00-10:20	Irena M. Jovanović	<i>Spectral distances of graphs</i>
10:20-10:40	Milan Bašić	<i>On the spread of integral circulant graphs</i>

Amfiteatar VII, predsedavajući: Đorđe Vučković

09:00-09:20	Snežana Gordić	<i>Stacionarni Kolombovi stohastički procesi sa primenama u rešavanju jednačina</i>
09:20-09:40	Limonka Koceva Lazarova	<i>Some compositions of distributions in neutrix calculus</i>
09:40-10:00	Marija Miteva	<i>Products of Distributions in Colombeau Algebra</i>
10:00-10:20	Daniel Velinov	<i>On the frequently hypercyclic C_0-semigroups</i>
10:20-10:40	Radoslav Božić	<i>The Application of Dynamic Software in the Examining Functions with Parameters and Their Derivatives</i>

10:40-11:00 pauza za kafu

Amfiteatar I, predsedavajući: Nenad Teofanov

11:00-11:20	Filip Tomić	<i>Paley-Wiener theorems and wave front sets</i>
11:20-11:40	Ivana Vojnović	<i>Continuity of pseudodifferential operators on mixed-norm Lebesgue spaces</i>
11:40-12:00	Đorđe Vučković	<i>Toroidal pseudodifferential operators in spaces of ultradistributions on \mathbb{T}^n</i>
12:00-12:20	Snježana Maksimović	<i>A sequential approach to ultradistribution spaces</i>

Amfiteatar VII, predsedavajući: Boriša Kuzeljević

11:00-11:20	Đorđe Baralić	<i>Small covers over neighborly polytopes</i>
11:20-11:40	Milan Zlatanović	<i>Koneksije na nesimetričnim Rimanovim mnogostrukostima</i>
11:40-12:00	Nenad O. Vesić	<i>IMPOSSIBLE: Whether, Why, How</i>
12:00-12:20	Jovana Nikolić	<i>Parcijalni kvazimorfizmi na grupi Hamiltonovih difeomorfizama kotangentnog raslojenja</i>
12:20-12:40	Anika Njamcul	<i>Maximal topologies obtained via ideals</i>

12:40-13:20

pauza za kafu i poslužnje

Amfiteatar I, predsedavajući: Tijana Ostojić

13:20-13:40	Stefan Ivković	<i>Semi-Fredholm theory on Hilbert C^*-modules</i>
13:40-14:00	Miloš Cvetković	<i>Decompositions of bounded linear operators</i>
14:00-14:20	Marija Cvetković	<i>Generalized Ulam-Hyers stability of integral and operator equations</i>
14:20-14:40	Nebojša Č. Dinčić	<i>Solving the Sylvester Matrix Equation $AX - XB = C$ when $\sigma(A) \cap \sigma(B) \neq \emptyset$</i>
14:40-15:00	Bogdan Đorđević	<i>On some properties of singular Sylvester operator equations</i>

Amfiteatar VII, predsedavajući: Danijela Mitrović

13:20-13:40	Ivana Đurđev	<i>Sendvič polugrupe u lokalno malim kategorijama</i>
13:40-14:00	Edin Glogić	<i>On Kirchhoff index, Laplacian energy, number of spanning trees of graphs and their relations</i>
14:00-14:20	Dragan S. Rakić	<i>Partial orders based on generalized inverses and annihilators</i>
14:20-14:40	Dragan Jočić	<i>Some notes on distributivity equations and aggregation operations</i>
14:40-15:00	Dušan J. Simjanović	<i>Fuzzy Relation Equations and Fuzzy Rough Approximation Operators</i>

Amfiteatar I

15:00

Zatvaranje Kongresa

Plenarna predavanja

Distributed optimization and learning

DUŠAN JAKOVETIĆ

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Recently, there has been a strong interest and progress in distributed optimization and learning, motivated by applications in IoT networks, distributed learning, and big data analytics. A typical studied setup assumes a generic, connected network of agents, whereby each agent i holds a private, convex cost function, known only by i , and the goal is to minimize the overall sum of the agents' costs subject to a (vector) variable of common interest. For the above problem, several distributed algorithms have been recently proposed, including (sub)gradient, augmented Lagrangian, and alternating direction methods. In this lecture, we present some of state-of-the-art methods, explain how they are constructed, how we can analyze their convergence and convergence rates, and present techniques on how the methods can be accelerated. We illustrate the algorithms on several real world applications and distributed computing environments.

Multilinear maps on multilinear varieties

LUKA MILIĆEVIĆ

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Let G_1, \dots, G_k be finite-dimensional vector spaces over a finite field \mathbb{F} . A *multilinear variety of codimension d* is a subset of $G_1 \times \dots \times G_k$ defined as the zero set of d forms, each of which is multilinear on some subset of the coordinates. A map ϕ defined on a multilinear variety B is *multilinear* if for each coordinate d and all choices of $x_i \in G_i$, $i \neq d$, the restriction map $y \mapsto \phi(x_1, \dots, x_{d-1}, y, x_{d+1}, \dots, x_k)$ is linear where defined. We show that a multilinear map defined on a multilinear variety of codimension d coincides on a multilinear variety of codimension $d^{O(1)}$ with a multilinear map defined on the whole of $G_1 \times \dots \times G_k$. This is joint work with Tim Gowers.

Iterative methods and neural networks for the computation of generalized inverses

MARKO D. PETKOVIĆ

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Generalized inverses of matrices are natural generalization of the ordinary matrix inverse for non-square and/or singular matrices. They have many different applications in various fields, from mathematical statistics and the least squares problem, up to control theory and image processing. We consider a general matrix iterative method of the type $X_{k+1} = X_k p(AX_k)$ for computing

generalized inverses ($p(x)$ is an arbitrary polynomial). These methods are direct generalization of well-known Schultz and hyperpower iterative methods for matrix inversion. Several new efficient high order iterative schemes of that type are proposed. Finally, a neural networks approach for the computation of generalized inverses is considered and demonstrated on few examples. This approach is particularly important in control theory and robotics, for solving the inverse kinematic problem.

Quasi-analytic representation theory of $(\mathbb{R}^d, +)$ over quasi-complete locally convex spaces

BOJAN PRANGOSKI

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In [1], Dixmier and Malliavin addressed the following problem. Given a Banach space E , let (π, E) be a representation of a real locally compact Lie group G , i.e. $\pi : G \rightarrow GL(E)$ is a homomorphism such that the mapping $G \times E \rightarrow E$, $(g, e) \mapsto \pi(g)e$, is continuous. Such representation induces a continuous action Π of the algebra $\mathcal{D}(G)$ on E given by

$$\Pi(f)e = \int_G f(g)\pi(g)edg, \quad f \in \mathcal{D}(G), \quad e \in E,$$

and it restricts to a continuous action to the Banach space of smooth vectors E^∞ consisting of all elements $e \in E$ for which the orbit maps $g \mapsto \pi(g)e$, $G \rightarrow E$, are smooth (thus the smooth vectors associated to such a representation are a $\mathcal{D}(G)$ -module). Dixmier and Malliavin proved that $E^\infty = \text{span}(\Pi(\mathcal{D}(G))E^\infty) = \text{span}(\Pi(\mathcal{D}(G))E)$; that is, the category of modules E^∞ over the algebra $\mathcal{D}(G)$ satisfies the weak factorisation property. Only recently the analytic variant of this problem was addressed. Namely, by denoting E^ω the space of elements of E whose orbit maps are analytic, the problem of interest here reads: does $E^\omega = \text{span}(\Pi(\mathcal{A}(G))E^\omega) (= \text{span}(\Pi(\mathcal{A}(G))E))$ hold, where $\mathcal{A}(G)$ is the space of analytic vectors of the (left) regular representation of an appropriate algebra and with it E^ω becomes an $\mathcal{A}(G)$ -module? The problem was affirmatively answered by Lienau [3] when $G = (\mathbb{R}, +)$, E is a Banach space and π is a bounded representation. This result was improved by Gimperlein, Krötz, and Lienau [2] by allowing E to be a Fréchet space without the restriction on the boundedness of the representation for general locally compact real Lie group. They obtained the weak factorisation property and $\text{span}(\mathcal{A}(G) * \mathcal{A}(G)) = \mathcal{A}(G)$; in their result $\mathcal{A}(G)$ is the space of analytic vectors of the regular representation of an appropriate convolution algebra.

In this talk we generalise the above result in the following two ways when $G = (\mathbb{R}^d, +)$:

(I) We allow E to be a general quasi-complete locally convex space.

(II) We will solve the problem in the general quasi-analytic case. Namely, we will define the space of ultradifferentiable vectors of Beurling and Roumieu type. Next, we will identify the appropriate convolution algebra over which the space of ultradifferentiable vectors will become a module. Finally, we will show that this category satisfies the factorisation property (without “span”).

The talk is based on collaborative works with Andreas Debrouwere and Jasson Vindas.

Literatura

- [1] J. Dixmier, P. Malliavin, *Factorisations de fonctions et de vecteurs indéfiniment différentiables*, Bull. Sci. Math. **102**(2) (1978), 307-330

- [2] H. Gimperlein, B. Krötz, C. Lienau, *Analytic factorization of Lie group representations*, J. Funct. Anal. **262** (2012), 667-681
- [3] C. Lienau, *Analytic representation theory of $(\mathbb{R}; +)$* , J. Funct. Anal. **257** (2009), 3293-3308

Predavanja

O jako palindromičnim rečima: ternarni slučaj

KRISTINA AGO

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Ova prezentacija je posvećena takozvanoj *MP-razmeri* date reči. Taj pojam je inicijalno bio definisan samo za binarne reči kao količnik dužine najkraće minimalno palindromične reči koja sadrži zadatu reč kao faktor, i dužine same zadate reči. Pritom se minimalno palindromična reč definiše kao reč koja ne sadrži palindromsku podreč dužu od polovine svoje dužine (jasno, svaka binarna reč sadrži palindromsku podreč dugačku bar toliko, što opravdava naziv). Pokazano je ranije da je u binarnom slučaju ta razmera najviše 4 i da se ova gornja granica ne može poboljšati, a ostalo je otvoreno pitanje da li se MP-razmera (tj. njen očigledan prirodan analogon) može definisati i u slučaju alfabeta neke arnosti veće od 2. U ovom izlaganju pokazaćemo da to jeste slučaj za ternarni alfabet, utvrdićemo da je najbolje gornje ograničenje MP-razmere u tom slučaju 6 i konstatovaćemo da se niz vrlo neintuitivnih osobina ranije uočenih u binarnom slučaju prenosi i na veću arnost.

Ovo je zajednički rad sa Bojanom Bašićem.

Frames for Hilbert and Banach spaces

SUZANA ALEKSIĆ

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We start the talk with brief introduction to Hilbert space frames. A frame is a generalization of a basis and allows us to reconstruct vectors in Hilbert space in a numerically stable way from its frame coefficients. Further, we give a straightforward generalization of frames for a Hilbert space to general Banach spaces. We characterize Banach frames in separable Banach spaces, and relate them to series expansions in Banach spaces but also show that Banach frames do not share all properties of Hilbert space frames. As an illustration of some results, we consider shift-invariant spaces and show an equivalence between the concept of p -frames, Banach frames, and the closedness of the space they generate. Since frames are very useful in signal processing, we discuss some modern techniques for sampling and reconstruction of functions in shift-invariant spaces.

Algoritmi za ostatke levog faktorijela i Kurepina hipoteza

VLADICA ANDREJIĆ

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Kurepina hipoteza kaže da nijedan neparan prost broj p ne deli svoj levi faktorijel koji definišemo sa $!p = 0! + 1! + \dots + (p-1)!$. Koristili smo nove tehnike optimizacije i izračunali ostatke od $!p$ pri deljenju sa p za svaki prost $p < 2^{40}$, čime smo pokazali da do te granice nema kontraprimera. Posmatrali smo i uopšten levi faktorijel koji je definisan sa $!^k n = (0!)^k + (1!)^k + \dots + ((n-1)!)^k$, gde smo pokazali da za svaki prirodan $1 < k < 100$ postoji neparan prost p koji deli $!^k p$. Dodatno smo izučavali egzistenciju prostih $p > 5$ za koje su svi ostaci od $2!, 3!, \dots, (p-1)!$ po modulu p različiti i pokazali da oni ne postoje za $p < 2^{40}$.

Literatura

- [1] V. Andrejić, M. Tatarevic, Searching for a counterexample to Kurepa's conjecture, *Math. Comp.* **85** (2016), 3061–3068.
- [2] V. Andrejić, M. Tatarevic, On distinct residues of factorials, *Publ. Inst. Math.* **100** (2016), 101–106.
- [3] V. Andrejić, A. Bostan, M. Tatarevic, Improved algorithms for left factorial residues, *arXiv:1904.09196 [math.NT]* (2019).

Obećanja svode konačne probleme na beskonačne

KRISTINA ASIMI

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Mnogi realni problemi se mogu predstaviti u obliku problema zadovoljenja ograničenja (CSP - Constraint Satisfaction Problem). Jedna od generalizacija CSP-a je PCSP (Promise CSP). Svi konačni PCSP problemi za koje trenutno znamo da su rešivi u polinomnom vremenu mogu se svesti na CSP probleme rešive u polinomnom vremenu. Ja dajem grupu PCSP problema za koje taj CSP mora biti beskonačan (pod pretpostavkom $P \neq NP$).

Characterization of wave front sets via Stockwell transform

SANJA ATANASOVA

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The talk is dedicated to resolution of wave front sets using the Stockwell transform defined with respect to the rotation group. The main results give the principles for directional smoothness, by providing criteria for regular directed points using the Stockwell transform. We work on the cases when the dimension is 1, 2, 4 and 8. Then, we define the Stockwell transform on arbitrary dimension, and obtain Parseval relation and inversion formula. At the end, we generalize our results for arbitrary dimension. This is a joint work with Stevan Pilipović, Bojan Prangoski and Katerina Saneva.

Small covers over neighborly polytopes

DORĐE BARALIĆ

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A simplicial n -polytope is called neighborly when every $\lfloor \frac{n}{2} \rfloor$ of its vertices span a face. Beside simplex the most known example of a neighborly polytope is a cyclic polytope. Hasui showed that the duals of cyclic polytopes in general, do not arise as the orbit space of quasitoric manifolds. In contrast to this result from toric topology, using computer search we show that in dimensions 4, 5, 6 and 7 there are plenty of examples of such manifolds. We succeeded to classify small cover over them and verify that the lifting conjecture holds for them. This is a joint work with Lazar Milenković.

MILAN BAŠIĆ

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The integral circulant graph $ICG_n(D)$ has the vertex set $Z_n = \{0, 1, 2, \dots, n-1\}$ and vertices a and b are adjacent if $\gcd(a-b, n) \in D$, where $D \subseteq \{d : d \mid n, 1 \leq d < n\}$. The spread of a graph is defined to be the difference between the greatest eigenvalue and the least eigenvalue of the adjacency matrix of the graph. Characterizing such graph(s) with maximum and minimum spread of fixed order is difficult. However, the problem is still interesting if we restrict the discussion to some special classes of graphs. In fact we solve this problem for the class of integral circulant connected graphs of prescribed order. Moreover, the maximum spread problem is in close relationship with the minimum least eigenvalue problem in a given class of graphs. Therefore, we determine the unique graph with minimum least eigenvalue among all connected $ICG_n(D)$ of prescribed order n .

The Application of Dynamic Software in the Examining Functions with Parameters and Their Derivatives

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The examining functions presents an important teaching content in the middle and higher education. The experience, as well as the previous research, has shown that a significant number of students have difficulties with learning and understanding this teaching content. The use of appropriate software enables the process of examining functions to be more effective, allowing more functions to be examined in a short time. Most of the software packages that can be used in examining functions allow the simultaneous observation of different representations of functions, primarily algebraic and graphical, as well as dynamically connecting these representations and forming a manipulative representation. Dynamic software, also, significantly simplifies the examining functions with parameters. The examining these functions without the use of dynamic software is time consuming and difficult for many students. The dynamic properties of the software packages such as GeoGebra allow, when examining the properties of a function with parameters, to change the value of each parameter individually and to observe changes within the graphical and algebraic representation of the function, or its properties. In this way it is possible to examine the influence of certain parameters on the properties of a function, as well as to notice differences between functions of the same class and similarities among the functions which belong to the different classes. Besides the mentioned, the application of dynamic software enables simultaneously observing the multiple representations of the functions and their derivatives, which contributes to more simplify comparison their properties and understanding the interdependence of the properties of functions and their derivatives.

Generalized Ulam-Hyers stability of integral and operator equations

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Recently obtained Perov type results inspired a new approach and generalization regarding Ulam-Hyers stability problems. Generalized Ulam-Hyers stability is previously defined by introducing a function, but in this case it could include an operator on a Banach space. The emphasis is on operator and integral equations. The application of different fixed point theorems of Perov type will be presented, as well as a comparison with well-known Ulam-Hyers stability results of these classes of equations.

Decompositions of bounded linear operators

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We give necessary and sufficient conditions under which a bounded linear operator T can be represented as the direct sum of a nilpotent (quasinilpotent) operator T_N and an operator T_M which belongs to any of the following classes: upper (lower) semi-Fredholm operators, Fredholm operators, upper (lower) semi-Weyl operators, Weyl operators, upper (lower) semi-Browder operators, Browder operators, bounded below operators, surjective operators and invertible operators.

Encoding Compensable Processes Into Adaptable Processes

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Recent IT advances (e.g., cloud computing and data analytics) are supported by large computing infrastructures, such as data-centers. Over the last few decades, infrastructures such as those supporting high performance computing, have grown in scale and complexity. Their power, flexibility and convenience goes hand-by-hand with the need of efficient energy consumption. Large systems may experience a variety of faults and/or errors with increasing frequency. We need new approaches to better develop the software that runs on large computing infrastructures: they should ensure that failures in applications are handled efficiently, so as to avoid wasting resources, especially energy. Against this background, many software applications are based on *long-running transactions* (henceforth LRTs). Frequently found in service-oriented systems, [1], LRTs describe time-extensive activities that involve several distributed components. For LRTs management, handling failures is one sensitive aspect: mechanisms for detecting failures and bringing the LRT back to a consistent state need to be explicitly programmed. The last decade has seen the emergence of specialized constructs used by LRTs, such as *exceptions* and *compensations*, which offer direct programming support. We had researched *compensation mechanisms*, [2], which are meant to compensate the fact that an LRTs has failed or has been aborted. Upon reception of an abortion or failure signal, compensation mechanisms are expected to install and activate alternative behaviors for recovering

system consistency. Related but different, a calculus of *adaptable processes* is a process calculus approach which specify the dynamic evolution of interacting systems, [3]. It is intended as a way of overcoming the limitations that process calculi have for describing patterns of dynamic evolution. Therefore, adaptable processes specify forms of dynamic reconfiguration which are triggered by exceptional events, not necessarily catastrophic.

Rather than developing new languages from scratch, one approach is to build on languages already developed for mobile, autonomic and service-oriented computing. Here we follow this approach, by formally connecting programming abstractions for compensation handling (typical of models for services and LRTs) and for run-time adaptation.

In particular, we compare mechanisms for *compensation handling* and *dynamic update* in calculi for concurrency. We have analyzed the *relative expressiveness* of these calculi: we develop *two encodings* of a process calculus with compensation handling into a calculus of adaptable processes. A (valid) encoding is a translation of processes of a source language into the processes of a target language that satisfies certain correctness criteria, which attest to the encoding's quality. The existence of a valid encoding shows that the target language is at least as expressive as the source language. Conversely, proving the non existence of such an encoding shows that the source language can express some behavior not expressible in the target language. By combining these positive and negative encodability results, differences in expressivity between languages can be established.

To define valid encodings, we rely on the abstract formulation defined in [4], focusing on *compositionality*, *name invariance*, and *operational correspondence* criteria. These three conditions correspond to two structural criteria and one semantic criterion. The structural criteria include compositionality and name invariance. Furthermore, structural criteria are needed in order to measure expressiveness of operators in contrast to expressiveness of terms. The semantic criterion, operational correspondence, is divided in *completeness* and *soundness* properties: the former ensures that the behavior of a source process is preserved by the translation in the target calculus; the latter ensures that the behavior of a translated (target) process corresponds to that of some source process.

Two encodings that we have developed differ in the target language considered: the first considers adaptable processes with *objective updates*, [5], in which a process is reconfigured by its context, whereas the second considers *subjective updates*, [6], in which, intuitively, a process reconfigures itself. Subjective updates turn out to be more efficient than objective ones in encoding primitives for compensation handling: the second encoding requires less computational steps than the first one to mimic a single computation step in the source language. Our encodings shed light on the (intricate) semantics of compensation handling and its key constructs. They also enable the transference of existing verification and reasoning techniques for adaptable processes to core languages with compensation handling.

Note: This talk is a result of joint work with Jovanka Pantović and Jorge A. Pérez.

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Analysis and numerical approximation of boundary value problems with fractional derivatives

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Over the last two decades, it has been shown that the fractional differential equations can be used in the modeling of various physical process that occur in viscoelastic media, disordered materials and subsurface flow. It is the reason why this subject has been attracting the attention of many researchers in recent years, and the interest is still growing.

We will consider a two-point boundary value problem whose highest-order term is a left fractional derivative of order $\alpha \in (1, 2)$ or composition of the left and the right fractional derivatives. We will give the analysis of the well-posedness of the weak formulation of proposed problems in suitable fractional-order Sobolev spaces. A finite difference scheme approximating the problem will be proposed and its stability will be investigated. Numerical examples, confirming the expected behavior of the method, will be presented.

Modulation spaces related to translation-invariant Banach spaces of quasi-analytic ultradistributions

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We define and study a new class of translation-modulation invariant Banach spaces of quasi-analytic ultradistributions. These new spaces show a certain stability under Fourier transform, duality and tensor product. Multiplication of the Fourier Lebesgue spaces $L^1_{\mathcal{D}}$ with elements from these spaces, also multiplication of elements from this space with elements from its dual are considered. We associate a new Banach space \mathcal{M}^F to translation-modulation invariant Banach space F . These space \mathcal{M}^F remains translation-modulation invariant Banach space. The duals of \mathcal{M}^F are also considered. The new defined spaces \mathcal{M}^F and results concerning them are generalizations of already known Modulation spaces of (ultra)distributions and results about them.

Solving the Sylvester Matrix Equation $AX - XB = C$ when $\sigma(A) \cap \sigma(B) \neq \emptyset$

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The method for solving the Sylvester equation $AX - XB = C$ in complex matrix case, when $\sigma(A) \cap \sigma(B) \neq \emptyset$, by using Jordan normal form is given, and the approach via Schur decomposition is presented. Also, the least-squares minimum-norm solutions were considered.

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2. B. D. Đorđević and N. Č. Dinčić, *Classification and Approximation of Solutions to the Sylvester Equation* (accepted)

On some properties of singular Sylvester operator equations

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This talk concerns the general solution of Sylvester operator equation and classification of its solutions. The set of the said solutions forms an operator algebra, which simultaneously allows solvability of some basic operator equations. As an application, connection with singular reaction-diffusion pdes is established.

L^p and almost sure convergence of an approximate method for stochastic differential equations

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In this paper an approximate method for solutions of stochastic differential equations is presented. The method considers Taylor expansion of the drift and diffusion coefficients of an equation with coefficients which do not necessarily satisfy the Lipschitz and linear growth conditions. Instead, they satisfy polynomial condition. L^p and a.s. convergence of the approximate solutions are proved and few examples are presented.

A stochastic analysis of the impact of fluctuations in the environment on pre-exposure prophylaxis for HIV infection

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We propose a stochastic model for HIV/AIDS transmission where pre-exposure prophylaxis (PrEP) is considered as a prevention measure for new HIV infections. A white noise is introduced into the model, representing fluctuations in the environment that manifest themselves on the transmission coefficient rate. We prove existence and uniqueness of a global positive solution of the stochastic model, and establish conditions under which extinction and persistence in mean hold. Numerical simulations are provided which illustrate the theoretical results and conclusions are derived on the impact of the fluctuations in the environment on the number of the susceptible individuals that are under pre-exposure prophylaxis.

Generalized solution to multidimensional cubic Schrodinger equation with delta potential

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We address the Cauchy problem for the defocusing cubic Schrodinger equation in 2D and 3D and the equation with a delta well potential in 3D. Solutions belong to the Colombeau algebra of generalized functions \mathcal{G}_{C^1, H^2} . Physically significant homogeneous problem in 2D and 3D has not yet been treated in this framework, whereas no classical results exist on the equation with delta potential.

Existence and uniqueness in appropriate spaces of generalized functions are proved. Compatibility with the classical H^2 solution is shown for the equation without potential. New estimates of second order derivatives are obtained for the regularized delta potential equation.

This is a joint work with Marko Nedeljkov.

Sendvič polugrupe u lokalno malim kategorijama

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Neka su X i Y fiksirani, različiti, neprazni skupovi, a $f, g : X \rightarrow Y$ proizvoljne funkcije. Jasno, f i g se ne mogu direktno komponovati; međutim, ako odaberemo proizvoljno preslikavanje $a : Y \rightarrow X$, rezultat kompozicije fag je funkcija $X \rightarrow Y$. Štaviše, skup svih funkcija iz X u Y , \mathcal{T}_{XY} , zajedno sa operacijom definisanom sa $f \circ g = fag$ za sve f, g iz \mathcal{T}_{XY} , je polugrupa. Nazivamo je sendvič

polugrupa funkcija. Prirodno je zapitati se pod kojim uslovima (odnosno za kakve "objekte") opisana ideja sendvič množenja funkcioniše i koje su osobine tako definisane polugrupe.

U tu svrhu definišemo sendvič polugrupu u lokalno maloj kategoriji. U predavanju sažeto izlažemo izabrane rezultate tri rada. Kombinujući njihov sadržaj, opisujemo osobine sendvič polugrupe u lokalno maloj kategoriji, a onda posmatramo i iste osobine na primeru sendvič polugrupe funkcija. Osobine koje koristimo da opišemo te polugrupe su karakterizacije Grinovih relacija, klasa i njihovih odnosa, strukturni opis regularne potpolugrupe, uz rezultate vezane za pitanja generisanja i ranga polugrupe.

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Some new fixed point results for convex contractions in B-metric spaces

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The various results for convex contraction mappings defined on the B-metric spaces are considered in this paper. Some recent results from the context of convex contractions defined on an ordinary metric spaces are generalized, extended, corrected and enriched to the ones on the so-called B-metric spaces with the coefficient $s \geq 1$. An example is given to support the obtained results.

On Kirchhoff index, Laplacian energy, number of spanning trees of graphs and their relations

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Let G be a simple connected graph with degree sequence (d_1, d_2, \dots, d_n) where $\Delta = d_1 \geq d_2 \geq \dots \geq d_n = \delta > 0$ and let $\mu_1 \geq \mu_2 \geq \dots \geq \mu_{n-1} > \mu_n = 0$ be the Laplacian eigenvalues of G . The Kirchhoff index, Laplacian energy and the number of spanning trees of G are defined as $Kf(G) = n \sum_{i=1}^{n-1} \frac{1}{\mu_i}$,

$LE(G) = \sum_{i=1}^n \left| \mu_i - \frac{2m}{n} \right|$, $\tau(G) = \frac{1}{n} \prod_{i=1}^{n-1} \mu_i$, respectively. In this paper we consider relation between $Kf(G)$ and $LE(G)$, and relation between $Kf(G)$ and $\tau(G)$. Some new lower bounds for $Kf(G)$ and $LE(G)$ are also obtained.

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Kolomboovi stohastički procesi sa vrednostima u $L^2(\Omega)$ su izučavani. Stacionarni Kolomboovi stohastički procesi su definisani i data je njihova karakterizacija. Gausovska stacionarna rešenja za neke klase stohastičkih parcijalnih diferencijalnih jednačina su analizirana u okvirima Kolomboovih stohastičkih procesa.

Napomena: Izlaganje je zasnovano na zajedničkom radu sa Mihaelom Obergugenbergerom, Stevanom Pilipovićem i Dorom Seleši.

O nekim aritmetički interesantnim kolekcijama permutacija

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U prvom delu izlaganja bavimo se problemom generisanja takozvanih savršenih nizova i uvodimo pojam optimalnih generatora perioda p , gde je p prost broj. Značajan otvoren problem je karakterizacija svih optimalnih generatora perioda p i u cilju njegovog rešavanja predstavljamo tri ekvivalentna problema prevedenih u jezik permutacija. Zatim ovaj problem uopštavamo na periode složenih dužina i stižemo do kolekcija permutacija za koje važi da je razlika svaka dva člana kolekcije takođe permutacija. Ispostavlja se da su ove kolekcije ekvivalentne kolekcijama ortogonalnih ortomorfizmima o kojima se ne zna mnogo. Na kraju ovog dela dajemo pregled do sada poznatih rezultata iz ove oblasti.

U drugom delu izlaganja bavimo se srodnim problemom: kolekcijama permutacija za koje važi da je zbir svaka dva člana kolekcije takođe permutacija. Posebno nas interesuju donja i gornja ograničenja kardinalnosti ovih kolekcija u zavisnosti od dužine permutacije. Dajemo konstrukciju koja za slučaj složene dužine permutacija znatno poboljšava dosadašnje rezultate.

Ovo je zajednički rad sa dr Bojanom Bašićem.

Semi-Fredholm theory on Hilbert C^* -modules

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We establish the semi-Fredholm theory on Hilbert C^* -modules as a continuation of Fredholm theory on Hilbert C^* -modules established by Mishchenko and Fomenko. We give a definition of a semi-Fredholm operator on Hilbert C^* -module and prove that these semi-Fredholm operators are those that are one-sided invertible modulo compact operators, that the set of proper semi-Fredholm operators is open and many other results that generalize their classical counterparts.

Next, given an \mathcal{A} -linear, bounded, adjointable operator F on the standard module $H_{\mathcal{A}}$; we consider the operators of the form $F - \alpha 1$ as α varies over $Z(A)$ and this gives rise to a different kind of spectra of F in $Z(A)$ as a generalization of ordinary spectra of F in the field of complex numbers. Using the generalized definitions of Fredholm and semi-Fredholm operators on $H_{\mathcal{A}}$ given by Mischenko and Ivkovic together with these new, generalized spectra in $Z(A)$ we obtain several results as a generalization of the results from the classical spectral semi-Fredholm theory given in papers by Zemanek, Djordjevic etc...

Finally we consider \mathcal{A} -Fredholm and semi- \mathcal{A} -Fredholm operators on Hilbert C^* -modules over a W^* -algebra \mathcal{A} . Using the assumption that \mathcal{A} is a W^* -algebra (and not an arbitrary C^* -algebra) we obtain several special properties such as that a product of two upper (or lower) semi- \mathcal{A} -Fredholm operators with closed image also has closed image, such as a generalization of Schechter-Lebow characterization of semi-Fredholm operators and a generalization of "punctured neighbourhood" theorem, as well as some other special results that generalize their classical counterparts.

Some notes on distributivity equations and aggregation operations

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The process of combining and merging several numerical values into a single representative one is called aggregation. A mathematical function performing this process is nondecreasing in each variable, fulfills the boundary conditions, and is called aggregation operation. Due to their simple nature, aggregation functions play important role in a large number of areas and disciplines from mathematics and natural sciences to economics and social sciences. In addition to research of applications of aggregation operations in many fields, during the last decades there exists an increasing interest in a theoretical study [2], which can lead to new possibilities of applications. Of the special interest is the study of additional properties of aggregation operations, which are usually derived from solutions of functional equations involving this kind of functions. Some of those properties are idempotency, associativity, modularity, migrativity, distributivity, etc. Investigation of the problem of distributivity has roots in [1] and, recently, it has been directed towards finding solutions for many known classes of aggregation operations. This paper is based on [3, 5], where a characterization of pairs of aggregation operations that are satisfying distributivity law, on both whole and restricted domain, is given. The focus is on the case when one function belongs to the class of aggregation operations with an absorbing element, and another one is from the class of aggregation operations with a neutral element. Therefore, the main concern is how to solve functional equations

$$F(x, G(y, z)) = G(F(x, y), F(x, z)), \quad x, y, z \in [0, 1]$$

and

$$F(x, G(y, z)) = G(F(x, y), F(x, z)), \quad x, y, z \in [0, 1], \quad \text{such that } G(y, z) < 1$$

where F is a T -uninorm in U_{\max} or a nullnorm, and G is a t -conorm or a uninorm from the class $U_{\min} \cup U_{\max}$.

It turned out that the distributivity law is a rather strong condition, since it simplifies the structure of the inner operator considerably, i.e., it is being reduced to an idempotent operator. On the other hand, the conditional distributivity, i.e., distributivity law on the restricted domain, produces a larger variety of solutions. The significance of the considered topic follows not only from the theoretical point of view, but also from its applicability in the utility theory for modelling some specific problems, e.g. problems with a previously imposed threshold [4].

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Spectral distances of graphs

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How similar two graphs can be? The natural and positive answer to this question is: when the two graphs are isomorphic. But, the *graph isomorphism problem* is NP-complete, which results in designing various heuristics to approximately solve it. In doing so, one actually creates the so called *graph similarity measures*. If this measures are defined by use of the eigenvalues and the eigenvectors of some specific graph matrices, we will say that they are spectrally based. But, while one explores the topological similarity of graphs using the similarity measures that may be spectrally characterized, that, on the other hand, the spectral similarity of graphs may be investigated by use of the entirely spectral measures. One such measure is the Manhattan *spectral distance* of graphs.

Let G_1 and G_2 be two non-isomorphic simple graphs of order n , and let $m_1(G_i) \geq \dots \geq m_n(G_i)$, $i = 1, 2$ be the spectra of these graphs with respect to some graph matrix M . The M -*spectral distance* $\sigma_M(G_1, G_2)$ of G_1 and G_2 is defined as [5]: $\sigma_M(G_1, G_2) = \sum_{i=1}^n |m_i(G_1) - m_i(G_2)|$. In that way, the graphs G_1 and G_2 are ϵ_M -*cospectral* if $\sigma_M(G_1, G_2) \leq \epsilon$, where ϵ is an arbitrary non negative real. Thus defined spectral distance is considered with respect to the adjacency matrix A , the Laplacian L and the signless Laplacian matrix Q .

In this talk, some of the relations related to the spectral distances of graphs with respect to the various graph matrices will be presented. The change of the spectral distance under some graph transformations will be exposed, as well as the results regarding the spectral distances between the regular graphs and the graphs that are obtained from them by applying some graph operations. The attention will be paid to the conjectures about the spectral distances with respect to the adjacency matrix. These conjectures are generated by use of the C++ program **SpecDist 2.0**. (see <http://poincare.matf.bg.ac.rs/~zstanic/sdist.htm>).

One of the problems originally exposed in [5] is related to the investigation of the cospectrality of graphs from special classes of graphs. If \mathcal{G} is an arbitrary set of graphs of order n , then the M -*cospectrality* $cs_{\mathcal{G}}^M(G)$ of $G \in \mathcal{G}$ is defined as follows: $cs_{\mathcal{G}}^M(G) = \min\{\sigma_M(G, H) : H \in \mathcal{G}, H \neq G\}$. Therefore, it is $cs_{\mathcal{G}}^M(G) = 0$ if and only if G has a cospectral mate. In the talk, the cospectrality of graphs from the family of graphs that is constructed by transforming the complete multipartite graph will be described.

The presented results are the joint work with Zoran Stanić (see [1], [2], [3] and [4]).

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The Strongest Inverted Compressed Column

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We study optimal shape of an inverted elastic column with concentrated force at the end and in the gravitational field. We generalize earlier results on this problem in two directions. First we prove a theorem on the bifurcation of nonlinear equilibrium equations for arbitrary cross-section column. Secondly we determine the cross-sectional area for the compressed column in the optimal way. Variational principle is constructed for the equations determining the optimal shape and two new first integrals are constructed that are used to check numerical integration. The classical Lagrange problem follows as a special case.

Kripkeove semantike za lambda račun sa parovima i sumama

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Matematička logika ima sve veći uticaj na razvoj računarstva i vještačke inteligencije. Posljednjih godina interes istraživača za izučavanje i razvoj vještačke inteligencije i mnogih drugih oblasti računarstva je značajno porastao, te se javila i potreba za izučavanjem već postojećih logičkih sistema, ali i formalizacijom novih. Jedna od programskih paradigmi, koja je postala predmet interesovanja mnogih istraživača, jeste probabilističko programiranje. Rezonovanje o vjerovatnoći je sve prisutnije u mašinskom učenju, robotici, vještačkoj inteligenciji, pa je prirodno da se javila potreba za matematičkom formalizacijom probabilističkih programskih jezika i njihovih osobina.

Motivisani vezom tipiziranog lambda računa i intuicionističke logike ([2]) i logičkim sistemom koji formalizuje rezonovanje o vjerovatnoći iskaza u intuicionističkoj logici ([5]), želimo da formalizujemo sistem za rezonovanje o vjerovatnoći iskaza „term M ima tip σ ”. Ideja je predstavljena u [1]. Cilj nam je da definišemo semantike i aksiomatski sistem tako da važi saglasnost i potpunost aksiomatskog sistema i predloženih semantika. Prvi korak ka ovom cilju jeste saglasnost i potpunost tipskog sistema lambda računa i odgovarajućih semantika. Posmatramo lambda račun sa parovima i sumama.

Na osnovu Curry-Howardove korespondencije ovaj sistem je ekvivalentan sistemu prirodne dedukcije za intuicionističku iskaznu logiku ([2]). Poznat je rezultat o saglasnosti i potpunosti intuicionističke iskazne logike i Kripkeovih semantika ([3]). Prateći ideju Kripkeovih semantika za intuicionističku iskaznu logiku ([3]) i Kripkeovih semantika za lambda račun sa osnovnim tipskim sistemom ([4]), definišemo Kripkeove semantike za lambda račun sa parovima i sumama. Saglasnost lambda računa sa predloženim semantikama smo dokazali indukcijom po dužini izvođenja (tipiziranja). Za dokaz potpunosti koristili smo konstrukciju kanoničkog modela. Kanonički modeli su posebna klasa modela koja se najčešće definiše na osnovu nekog skupa tako da zadovoljava samo formule koje pripadaju tom skupu. Potpunost je direktna posljedica postojanja kanoničkog modela i gore navedene osobine.

Cilj je da proširimo ovaj sistema na sistem u kome ćemo moći zapisati rečenicu „vjerovatnoća da term M ima tip σ je veća ili jednaka $\frac{2}{3}$ ”. Osnova ovog sistema će biti lambda račun sa parovima i sumama za koji smo definisali Kripkeove semantike i dokazali saglasnost i potpunost tipiziranja i navedenih semantika.

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Some compositions of distributions in neutrix calculus

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In this presentation some new contributions in neutrix calculus will be presented. The obtained results refer to some pairs of distributions for which the composition and the convolution product can not be calculated in normal sense. The main objective is the application of the basic definitions in the neutrix calculus, in order to calculate composition and convolution product of distributions which not exist in usual sense. On this way the set of pairs of distributions for which these types of products can be calculated, is extended.

Existence and asymptotic behavior of q -regularly varying solutions of nonlinear second order q -difference Thomas-Fermi equation

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This paper deals with necessary and sufficient conditions for the existence of nonoscillatory solutions of nonlinear q -difference Thomas-Fermi equation

$$D_q(a(t)\Phi_\alpha(D_q(x(t)))) = b(t)\Phi_\beta(x(qt)), t \in q^{\mathbb{N}_0},$$

where $q > 1$ and $\Phi_\alpha(x) = |x|^\alpha \text{sgn} x$, with assumption that coefficients $a, b : q^{\mathbb{N}_0} \rightarrow (0, \infty)$ are q -regularly varying functions and $\alpha > \beta > 0$. Moreover, with the help of q -regular varying Karamata theory, precise asymptotic behavior of such solutions is described.

Distributed Fixed Point Methods for Solving Systems of Linear Algebraic Equations

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Linear systems of equations are present in various fields of applied mathematics, either coming directly from real-world problems, or appearing during optimization process. We consider a class of fixed point methods for solving linear systems of equations in distributed computer environment. Each node generates a sequence of approximations that converges to the solution under conditions that resemble classical results for solving linear systems. Numerical results with respect to the computational and communication costs are presented.

Stability of the Stochastically Perturbed Tumor-Immune Interaction Model with Delay

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The purpose of this talk is the analysis of stochastic model for tumor-immune interaction with delay. More precisely, the deterministic delay tumor-immune interaction model is extended by introducing random perturbations and the corresponding stochastic model is obtained. For such obtained model, the existence and uniqueness of the global positive solution is proven, and then, by using suitable Lyapunov functionals, the stability conditions for equilibrium state when tumor cells and resting cells approach their carrying capacities are obtained. The talk is closed by presenting numerical simulation with reliable data to verify theoretical results.

Numerical solution of transport equation by means of the method of characteristics

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Attention will be devoted to the numerical solution of the transport equation by the method of characteristics in one and two dimensions, as well as comparing it with analytical solutions and studying convergence according to discrete and continuous steady states. In the one dimensional numerical scheme, linear or approximations of a higher order of accuracy can be used. Also, we are interested in what is going on with the numerical solution when time goes to infinity. A continuous steady state is an exact solution after infinitely many time, and a discrete steady state is a numerical solution after infinitely many iterations. In two dimensional scheme we will consider mesh splitted in rectangles. Here we can use linear or the second order approximations. In the last part some of numerical simulations will be presented.

I present the results of working together with professors Bérénice Grec and Yohan Penel.

Globalna teorija ultrafiltera

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Rudin-Keisler uređenje je tipičan metod za poređenje filtera. Najviše rezultata dobijeno je posmatranjem ovog uređenja na posebnoj klasi ultrafiltera na prebrojivom skupu, P-tačkama. W. Rudin je uveo P-tačke pedesetih godina dvadesetog veka, kada je pokazao da Kontinuum hipoteza implicira njihovo postojanje. Na predavanju ćemo ukratko objasniti šta se znalo o Rudin-Keisler uređenju P-tačaka, a onda dati i nove informacije o ovom uređenju. Ovi rezultati dobijeni su u saradnji sa Dilipom Raghavanom i Jonathanom Vernerom.

Some majorization relations and their linear preservers on $\ell^p(I)$

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Recent results in the infinite-dimensional majorization theory are presented. Extensions of majorization relations using stochastic operators on the discrete Lebesgue space $\ell^p(I)$, $p \in [1, \infty)$ where I is an arbitrary non-empty set, are discussed. Concrete forms of linear preservers of these relations are given, when I is an infinite set.

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We introduce and analyze fundamental sequences of smooth functions partitioned into equivalence classes which we call s -ultradistributions. The spaces formed by these classes will be denoted as \mathcal{U}'^* . We prove the existence of an isomorphism between \mathcal{U}'^* and the space of ultradistributions of Beurling type in case $*$ = $(p!)^t$ and of Roumieu type in case $*$ = $\{p!\}^t$, $t > 1$.

Aproksimacioni algoritmi za minimizaciju uskog grla u asimetričnoj verziji problema trgovačkog putnika

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U ovom radu izučavamo minimizaciju uskog grla u asimetričnoj varijanti problema trgovačkog putnika, gde je cilj minimizovati težinu najveće ivice u turi, umesto sumu svih ivica kao u standardnoj varijanti problema. Iako usvajamo pretpostavku da težine ivica zadovoljavaju nejednakost trougla, one nisu nužno simetrične, tj. težina ivice od a do b ne mora biti ista kao težina ivice od b do a . Bez pretpostavke o nejednakosti trougla, ovaj problem je NP-težak za razumnu aproksimaciju. Ključni doprinosi ovog rada su $O(\log n)$ -aproksimacioni algoritam, kao i nekoliko aproksimacija do na konstantu za specijalne slučajeve problema.

Comparison of the Euler-Maruyama and backward Euler methods for a class of pantograph stochastic differential equations

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The existence, uniqueness and almost sure polynomial stability of solutions for pantograph stochastic differential equations are considered, under nonlinear growth conditions. The convergence in probability of the Euler-Maruyama solution is proved under the same nonlinear growth conditions. Adding the linear growth condition, we show that the almost sure polynomial stability of the Euler-Maruyama solution implies the almost sure polynomial stability of the exact solution. Moreover, convergence in probability on finite time intervals is established for the backward Euler method. Additionally, under certain more restrictive conditions, which do not include the linear growth condition on the drift coefficient of the equation, it is proved that these solutions are globally a.s. asymptotically polynomially stable. Numerical examples are provided in order to support theoretical results.

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Nonlocal boundary value problems have a great theoretical and practical significance. On the one hand, they represent interesting generalization of classical boundary value problems. On the other hand, they can serve as mathematical models of some physical phenomena related to energy and mass transfer in domains with layers. These mathematical models we call transmission or interface problems. In this paper we consider a class of non-standard elliptic transmission problem in disjoint domains. As a model example we take an area consisting of two non-adjacent rectangles. In each rectangle an Robin's boundary value problem is given. The interaction between their solutions is described using nonlocal integral conjugation conditions Robin-Dirichlet type. For the model problem the existence and uniqueness of its weak solution in appropriate Sobolev-like space is proved. A finite difference scheme approximating this problem is proposed and analyzed. An estimate of the convergence rate, compatible with the smoothness of the input data is obtained.

Intuicionističko zasnivanje matematike i primene u muzici, arhitekturi, obrazovanju. . .

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Tokom XIX i početkom XX veka, učinjen je napor da se matematika zasnjuje na diskretnom jeziku i formalnoj logici. U suštini formalističkog poduhvata se nalazi *eliminacija vremena*, kako je Emil Mejerson označio sklonost modernističkog rasuđivanja da različitnost i promenu svodi na istovetnost i stalnost. Gedelovi i Tjuringovi, a potom i Čejtinovi, rezultati ubrzo su ustanovili nepotpunost ovog svetonazora. S druge strane, intuicionizam smatra vreme osnovom svesti što znači da matematika predstavlja izgradnju vremenom sve složenijih struktura. Skelet matematike u tom pogledu čini vremenski kontinuum koji odgovara pojmu broja zasnovanom na postupku merenja. Sameravanje veličina Euklidovim algoritmom uspostavlja verižni razvoj čiji svaki član određuje cifru po cifru decimalnog zapisa skoro sigurno, u skladu sa Lohsovom teoremom.

Uviđanje mogućnosti da se pojedine strukture utope u kontinuum je netrivialan zadatak od suštinskog značaja za intuicionističko zasnivanje, čemu je posvećena Petrovićeva teorija matematičkih spektara. Po sredi je stav o jedinstvu matematike i njenih primena, koja se zasniva u analogiji sa svetlosnim spektrima u fizici ili pak fizičkoj hemiji. Značaj ove teorije za muziku se tiče kvintnog kruga koji se dosledno razvija u pojmovima vremenskog kontinuuma.

Po pitanju arhitekture, intuicionizam se ogleda u naučnoj estetici Milutina Borisavljevića koji ju je smatrao vremenskom umetnošću. Njegova teorija smatra pojam vremena suštinom arhitektonskog dizajna. Borisavljevićeva estetika je uspešno primenjena na antičke hramove, povezujući njihove ritmove i proporcije sa zakonitostima opažanja.

Primene u obrazovanju zahtevaju intuicionističku teoriju saznanja koja je izraz kognitivne hijerarhije. Povezanost broja i vremena predstavlja bitnu stavku koja bi bila kadra pospešiti matematičko obrazovanje, obelodanjujući njegov značaj u obrazovnom sistemu. Vremenski kontinuum pri tom

nadilazi narcističku kulturu modernizma, svedenu na ograničeno prisustvo koje podrazumeva ravnodušnost spram prošlosti i bezličan odnos spram budućnosti. Teorija je, prema tome, takođe značajna za humanističke nauke i njihovu metodologiju.

Products of Distributions in Colombeau Algebra

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The problem about product of two arbitrary distributions is one of the main problems that classical theory of distributions had come across. Many attempts have been done for overcoming this problem. The construction of the Colombeau algebra seems to be an optimal solution until now for dealing with products of distributions. Colombeau algebra is an associative, differential algebra and the space of Schwartz distributions is embedded in it. The most important feature of the Colombeau algebra is that the product of elements in it generalises the classical product of distributions, thus the classical product of two distributions, if it exists, and the new one obtained in Colombeau algebra (Colombeau product of distributions) are equal. Furthermore, in Colombeau algebra we can obtain many products of two singular distributions which in the classical theory are not defined. One of the advantages of Colombeau theory of generalized functions is that we can operate with singular distributions easily as well as with smooth functions.

I will present the idea for the construction of such algebra and some examples with results about products of two singular distributions that can not be calculated in the classical theory of distributions.

Emulaciona ekvivalencija kombinatornih igara

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Bavimo se imparcijalnim kombinatornim igrama. Uvešćemo pojam *emulacione ekvivalencije* dve takve igre: intuitivno govoreći, igre koje su emulaciono ekvivalentne „ponašaju se potpuno isto“ sa stanovišta mogućih poteza u odgovarajućim pozicijama.

Takođe ćemo uvesti jednu specijalnu igru na grafovima, i pokazaćemo da odgovarajućim odabirom njenih početnih parametara možemo dobiti igre koje su emulaciono ekvivalentne s čitavim nizom poznatih i široko izučavanih kombinatornih igara, kao što su Nim, tzv. igra oduzimanja, Chomp, Notakto itd. Međutim, pokazaćemo da naša igra na grafovima ipak nije „svemoćna“, u tom smislu što ćemo za određenu igru (izvesnu modifikaciju igre Nim) pokazati da ona nije emulaciono ekvivalentna s uvedenom grafovskom igrom (ni za kakav odabir parametara).

Ovo je zajednički rad sa Bojanom Bašićem i Nikolom Milosavljevićem.

Characterizing the nuclearity of Gelfand-Shilov spaces

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As nuclear spaces have proven themselves to be valuable tools in functional analysis, one may just look at validity of the Schwartz kernel theorem, it is of great importance to verify which well-known spaces meet the property. This is certainly true for spaces of ultradifferentiable functions and the talk will be dedicated to the characterization of nuclearity of certain Gelfand-Shilov spaces. More specific, we characterize the nuclearity of the Beurling-Björck spaces $\mathcal{S}_{(\eta)}^{(\omega)}(\mathbb{R}^d)$ and $\mathcal{S}_{\{\eta\}}^{\{\omega\}}(\mathbb{R}^d)$ in terms of the defining weight functions ω and η , as well as the nuclearity of the spaces $\mathcal{S}_{\mathcal{W}}^{(M_p)}(\mathbb{R}^d)$ and $\mathcal{S}_{\mathcal{V}}^{\{M_p\}}(\mathbb{R}^d)$, which cover the classical Gelfand-Shilov spaces, in terms of the defining weight sequence M_p and weight system \mathcal{W} or \mathcal{V} .

This talk is based on collaborative works with Andreas Debrouwere and Jasson Vindas.

Parcijalni kvazimorfizmi na grupi Hamiltonovih difeomorfizama kotangentnog raslojenja

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Na predavanju ćemo objasniti kako se u ambijentu kotangentnog raslojenja konstruišu konormalne spektralne invarijante koristeći Lagranževu Florovu homologiju. Koristeći ove spektralne invarijante definisaćemo parcijalne kvazimorfizme na grupi hamiltonovih difeomorfizama kotangentnog raslojenja. Rezultati koji će biti predstavljeni deo su zajedničkog rada sa Jelenom Katić i Darkom Milinkovićem.

Maximal topologies obtained via ideals

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In this talk I will present the idea of generating new topologies from a given topological space (X, τ) and an ideal \mathcal{I} on X , which was known since the work of Kuratowski [2]. Using the local function $A^* = \{x \in X : A \cap U \notin \mathcal{I} \text{ for each } U \in \tau(x)\}$ (see [1]), a new topology τ^* can be obtained such that $\tau \subseteq \tau^*$. This expansion, under certain conditions concerning τ or the given ideal, can share properties of the old topology. I will present our results about the specific ideals which generate the new topology in such a way that given sets can be made open in τ^* while simultaneously preserve regular open sets or connectedness. Also, we studied the existence of an ideal such that the resulting topology is the maximal one preserving a certain property. This will also be illustrated through some examples.

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A class of neutral stochastic differential equations with time-dependent delay and Markovian switching and the Euler-Maruyama approximation

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Subject of consideration is a class of neutral stochastic differential equations with time-dependent delay and Markovian switching. The existence and uniqueness, as well as stability results of the exact solution for a class of neutral stochastic differential equations with time-dependent delay and Markovian switching are considered. Moreover, the convergence in probability of the Euler-Maruyama solutions is established regardless whether or not the delay function is bounded. These results are obtained under certain non-linear growth conditions on the coefficients of the equation. Adding the linear growth condition on the drift coefficient, the almost sure exponential stability of the Euler-Maruyama solution is proved in the case of the bounded delay. It should be stressed that the neutral term is also hybrid, that is, it depends on the Markov chain. An example and numerical simulations are provided to illustrate the theoretical results.

On an Inexact Restoration Subgradient Method

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We consider unconstrained minimization of a finite sum of nondifferentiable convex functions. A new subgradient method, which employs suitable approximations of the objective function and subgradient, is proposed. The parameter which determines the expected number of average sample size for the function and gradient approximation is obtained by means of Inexact Restoration method. The convergence of the method is proved and numerical results that illustrate the benefits of the proposed method are presented.

Kinetic monatomic gas mixture models: on the Cauchy problem and L^p theory for the system of Boltzmann equations

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The lecture will focus on the analysis of kinetic systems for mixtures of monatomic gases with different masses. This corresponds to a Boltzmann system for the evolution of vector valued distribution function. The collision or interaction law, as much as the modelling of the transition probability rates for pairwise interactions, are crucial components in the dynamics.

We will present some recent rigorous properties developed for the multi-component monatomic gas system described by coupled Boltzmann equations corresponding to the dynamics of elastic mixing of particles characterized by their identical shapes (spheres) but different masses.

These results are obtained in collaboration with Irene M. Gamba and Erica De La Canal.

Anti-Gaussian quadrature rule for trigonometric polynomials

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We investigate an anti-Gaussian quadrature rule with maximal trigonometric degree of exactness with respect to an even weight function on $[-\pi, \pi)$. Its error is equal in magnitude but of opposite sign to corresponding Gaussian formula. We give the method for its construction based on relations between nodes and weights of the quadrature rule for trigonometric polynomials and those of the quadrature rule for algebraic polynomials which were given in [1]. Also, we introduce averaged Gaussian quadrature formula for trigonometric polynomials and, at the end, we give some numerical examples.

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Partial orders based on generalized inverses and annihilators

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Several matrix partial orders based on different types of generalized inverses have been developed during the past several decades. Some of them are minus, star, sharp and core partial order. This talk concerns extensions of these orders to the setting of rings, particularly von Neumann regular rings, Rickart rings and Rickart $*$ -rings. The setting of Banach and Hilbert space operators will be discussed as well.

Approximate solution to pressureless gas dynamics model and shadow wave tracking procedure

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Solutions to systems of hyperbolic conservation laws often develop discontinuities in finite time which leads to formation of shocks. Those solution are sometimes unbounded (singular) which is expressed through the appearance of Dirac delta function. In that case the classical methods for solving such systems are no longer useful. We develop the universal algorithm for constructing an approximate solution to general initial data problems which is based on solving the Riemann problems and tracking singular wave interactions. Use of the shadow waves as a singular solutions enable as to easily solve the interaction problems. We demonstrate the idea using the well known pressureless gas dynamics model and we show that the method can also be adapted to initial data problems containing delta function.

Fuzzy Relation Equations and Fuzzy Rough Approximation Operators

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Rough set theory and fuzzy set theory are two useful mathematical tools for dealing with uncertainty and granularity in information systems. Motivated by the studies of fuzziness and roughness in algebraic systems and partially ordered sets such as semigroups, rings and lattices, we introduced systems of fuzzy relation inequalities and equations whose one side consists of the composition of a given family of fuzzy relations and an unknown fuzzy set, and another one consist only of the same unknown fuzzy set. Equations of this type are known in the literature as eigen fuzzy sets equations. The systems are studied in the framework of a quantale as the structure of membership values, by means of fuzzy quasi-orders and closures on a collection of fuzzy sets. Also, algorithms for computing extremal solutions to these systems are provided.

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Hešov broj date geometrijske figure T (topološkog diska) definišemo kao maksimalan nenegativan ceo broj n takav da se (donekle neformalno rečeno) oko figure T može obmotati n slojeva sastavljenih od figura podudarnih sa T ; ako takav maksimalan broj ne postoji, definišemo da je Hešov broj jednak ∞ . Motivacija koja stoji iza ove definicije jeste da Hešov broj date figure meri koliko pomoću date figure možemo prići „blizu“ teselaciji čitavog prostora (što je Hešov broj veći, možemo prići „bliže“ teselaciji, a Hešov broj je ∞ ako i samo ako postoji teselacija prostora podudarnim kopijama date figure).

Gotovo svi dosadašnji radovi na temu Hešovog broja smešteni su u dvodimenzionalan prostor, tj. u ravan (najčešće euklidsku). Osnovno otvoreno pitanje u vezi s Hešovim brojem jeste da li skup konačnih vrednosti koje se mogu javljati kao Hešovi brojevi u \mathbb{E}^2 ima gornje ograničenje. Veruje se da taj skup jeste ograničen, a trenutni „rekord“ drži figura sa Hešovim brojem 5.

U ovom izlaganju izučavaćemo Hešov broj u višedimenzionalnim euklidskim prostorima. Pokazaćemo da, za svaku dimenziju d oblika 2^k , u prostoru \mathbb{E}^d postoji figura čiji je Hešov broj k . Specijalno, ovo znači da, za $d \rightarrow \infty$, skup konačnih vrednosti koje se mogu javljati kao Hešovi brojevi u \mathbb{E}^d nema gornje ograničenje.

Ovo je zajednički rad sa Bojanom Bašićem.

Stochastic competition model with herd behavior and Allee effect

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In this paper we study a competition model with herd behavior and Allee effect whose dynamics is described by the system of stochastic differential equations. We prove some significant properties, such as the existence-and-uniqueness and boundedness of positive solution. Moreover, the asymptotic stability in mean of the equilibrium states of the considered system, are obtained under some conditions.

Metric logics

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The problem of reasoning with uncertain knowledge is an ancient problem dating. The last decades there is a growing interest in the field connected with applications to computer science and artificial intelligence. Some of the proposed formalisms for representing, and reasoning with, uncertain knowledge are based on metric logics. We introduce and investigate a formal language that is an

extension of classical propositional language obtained by adding new binary (modal-like) operators of the form $D_{\leq s}$, $D_{\geq s}$, $s \in Q_0^+$ and $D_{\approx a}$, $a \in Q[\epsilon]$. Our language allows making formulas such as $D_{\leq s}(\alpha, \beta)$ with the intended meaning ‘distance between formulas α and β is less than or equal to s ’. The semantics of the proposed language consists of possible worlds with a distance function defined between sets of worlds. Our main concern is a complete axiomatization that is sound and strongly complete with respect to the given semantics.

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Logarithmic (translationally) rapidly varying sequences and selection principles

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Motivated by issues in the information theory (self-information of the system) we will investigate a class of logarithmic (translationally) rapidly varying sequences, which is a strict subclass of rapidly varying sequences. We will introduce some basic properties. Further, we will see a proof that the Rothberger’s and Kocinac’s selection principle hold, when this class is on the second coordinate, and on the first coordinate is the class of positive and unbounded sequences.

Paley-Wiener theorems and wave front sets

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In this talk we discuss different versions of Paley-Wiener theorems for subclasses of smooth functions. In particular, such functions can be characterized through the decay properties of their Fourier transforms. Moreover, we introduce the corresponding wave-front sets and discuss their basic properties.

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Multiple orthogonal polynomials (algebraic and trigonometric polynomials of semi-integer degree) are a generalization of orthogonal polynomials in the sense that they satisfy orthogonality conditions with respect to the $p \in \mathbb{N}$, $p \geq 2$, weight functions. Such polynomials in the algebraic case arise in the theory of simultaneous rational approximation, in particular in Hermite-Padé approximation of a system of $p \in \mathbb{Z}^+$ Markov functions, and they have wide range of applications, in rational approximation, number theory, random matrices, integrable systems and geometric function theory.

Starting with a problem that arise in the evaluation of computer graphics illumination models, Borges [Numer. Math. **67** (1994), 271–288] has examined the problem of numerically evaluating a set of p definite integrals taken with respect to distinct weight functions but related by a common integrand and interval of integration. Since for such integrals it is not efficient to use a set of p Gauss-Christoffel quadrature rules, optimal set of quadrature rules are introduced and multiple orthogonal algebraic polynomials are used for its characterization. Also, we consider multiple orthogonal trigonometric polynomials of semi-integer degree, which are necessary for construction of optimal set of quadrature rules with an odd number of nodes for trigonometric polynomials.

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Generalized Polynomial Chaos and Stochastic Galerkin Method

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Polynomial chaos expansion (and generalized polynomial chaos expansion) have been extensively used to solve a number of ODEs and PDEs that contain uncertainties, using well known orthogonal polynomials, such as Hermite, Jacobi, Laguerre, Legendre, etc. A stochastic Galerkin method will be

presented as a tool for numerical solving of some stochastic ordinary differential equations (SODEs) and stochastic partial differential equations (SPDEs). The main advantage of the Galerkin method is that it replaces a stochastic equation by a system of deterministic equations. The complexity of the obtained system depends on the order of the (generalized) polynomial chaos, and on the chosen governing stochastic equation. Numerical examples are presented to support the analysis.

Interakcija nestišljivog viskoznog fluida sa termoelastičnim pločama

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Na ovom predavanju ću pričati o našem nedavnom rezultatu u kojem smo se bavili fizičkim problemom interakcije između fluida i termoelastičnih ploča. Fluid ispunjava posudu čiji je poklopac termoelastična ploča i međusobno interaguju jedno sa drugim kroz vreme, stoga je domen fluida promenljiv (free boundary problem). Fluid je opisan trodimenzionalnim nestišljivim viskoznim Navier-Stokes jednačinama dok je ploča opisana sistemom od dve jednačine - balans sila i toplotna jednačina. Jednačina balansa sila je nelinearna i obuhvata više modela koji se pojavljuju u literaturi:

1. Semilinearan slučaj: Kirchhoff, von Kármán i Berger ploče (videti [1]);
2. Kvazilinearan slučaj: Nelinearnost u jednačini balansa sila ploče je oblika $\Delta((\Delta w)^3)$, gde je w vertikalni izmeštaj ploče (videti [2]);

Pokazaću postojanje slabog rešenja za ovaj problem konstruisanjem posebnog aproksimiranog problema čija rešenja konvergiraju ka rešenju originalnog problema. Aproksimirani problem je numeričke prirode i veoma jednostavno se rešava elementarnim alatima iz teorije običnih diferencijalnih jednačina dok se konvergencija rešenja zasniva na uniformnim ocenama energije. Ovaj rezultat je postignut zajedno sa mojim mentorom, profesorom Wang Ya-Gaungom sa istog fakulteta.

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On the frequently hypercyclic C_0 -semigroups

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The main subjects of this talk are f -frequently hypercyclic and q -frequently hypercyclic ($q \geq 1$), C_0 -semigroups defined on complex sectors in the setting of infinite-dimensional separable Fréchet spaces. Results on generalized frequently hypercyclic translation semigroups and generalized frequently hypercyclic semigroups induced by semiflows on weighted spaces will be presented. Several illustrative related applications, concerning the previous results, will be given.

IMPOSSIBLE: Whether, Why, How

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This is the story about impossible in differential geometry, generally in mathematics.

In the first section, it is recalled how most of my and my colleagues' scientific papers in this subject start.

In the second section, it is asked why the unique is unique and the impossible is impossible. After that, it is given the common simple answer and the author's simpler answer to the question WHY IT IS IMPOSSIBLE. The author also answered why unique geometrical objects are unique.

In the third section, it is discussed what maybe is one of many senses of the previously studied IMPOSSIBLE.

Key words: geometric mapping, affine connection, invariant, application

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Continuity of pseudodifferential operators on mixed-norm Lebesgue spaces

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Mixed-norm Lebesgue spaces found their place in the study of some questions in the theory of partial differential equations, as it can be seen from recent interest in continuity of certain classes of pseudodifferential operators on these spaces. We prove the boundedness of a large class of pseudodifferential operators and also the boundedness of integral operators on mixed-norm Lebesgue spaces. Some generalisations to mixed-norm Sobolev spaces are obtained as well, together with applications to some interpolation and compactness results.

This is a joint work with Nenad Antić and Ivan Iveć.

Toroidal pseudodifferential operators in spaces of ultradistributions on \mathbb{T}^n

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In this talk we will study a class of symbols and corresponding pseudodifferential operators of finite order on torus \mathbb{T}^n that act continuously on a space of ultradistributions on \mathbb{T}^n , of Beurling and Roumieu type, and develop symbolic calculus for these classes.

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Heroin spada u najopasnije droge današnjice, prema brzini razvoja zavisnosti koju izaziva. Istraživanja su pokazala da kombinacija genetskih, psiholoških i uticaja društvene sredine dovode do toga da osoba postane heroinski zavisnik. Širenje heroinske zavisnosti unutar populacije se može uporediti sa prenošenjem infektivne bolesti među stanovništvom. U ovom radu se razmatra stohastički heroinski model koji uzima u obzir sve faktore spoljašnje sredine koji dovode do toga da osoba postane zavisna od heroina. Primenom odgovarajućeg funkcionala Ljapunova dolazi se do uslova koje je neophodno da zadovoljavaju koeficijenti sistema da bi trivijalni ekvilibrijum bio stohastički stabilan, odnosno kada u populaciji nema heroinskih zavisnika. Sa druge strane, rešenje stohastičkog sistema oscilira tokom vremena oko ekvilibrijuma determinističkog sistema kada je prisutna upotreba heroina među stanovništvom. Zaključak je, da ukoliko se intenzivira sa merama suzbijanja zavisnosti, u prvom redu prevencije u najranijem uzrastu, broj osoba koje koriste heroin imaće trend pada.

O obogaćenom stepenom grafu grupe

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Usmereni stepeni graf grupe uveli su Kelarev i Quinn kao prost digraf čiji skup čvorova čini nosač grupe, a u kome $x \rightarrow y$ ako je y stepen elementa x , dok je *stepeni graf* grupe njemu odgovarajući neusmereni prost graf. *Obogaćeni stepeni graf* grupe, koji su uveli Cameron i ostali, je graf sa istim skupom čvorova takav da su x i y susedni ako su oni stepeni nekog elementa $z \in G$.

Cameron je dokazao da stepeni graf određuje usmereni stepeni graf. Mi dokazujemo da i obogaćeni stepeni graf određuje usmereni stepeni graf. Pokazujemo da je svaki izomorfizam između stepenih grafova konačnih grupa takođe i izomorfizam odgovarajućih obogaćenih stepenih grafova, i nalazimo sve konačne grupe čiji obogaćeni stepeni grafovi imaju grupu automorfizama sa određenim osobinama. Pored ovoga, opisujemo i obogaćene stepene grafove konačnih Abelovih grupa. Konačno, dajemo karakterizaciju konačnih nilpotentnih grupa čiji su enhanced power grafovi perfektni, i predstavljamo dovoljan uslov za slabu perfektnost obogaćenog stepenog grafa konačne grupe.

Ovo je na zajednički rad sa Ivicom Bošnjakom i Rozálijom Madarász.

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In this talk we will present the nonlinear stochastic evolution equations with Wick-type nonlinearities set in the framework of white noise analysis. In particular, these equations are of the form

$$\begin{aligned}u_t(t, \omega) &= A u(t, \omega) + \Phi^\diamond(u(t, \omega)) + f(t, \omega), \\u(0, \omega) &= u^0(\omega), \quad \omega \in \Omega,\end{aligned}$$

for $t \in (0, T]$, where $u(t, \omega)$ is an X -valued generalized stochastic process; X is a certain Banach algebra, A corresponds to a densely defined infinitesimal generator of a C_0 -semigroup and Φ^\diamond is given analytic function. These equations include the stochastic Fujita equation, the stochastic Fisher-KPP equation and the stochastic FitzHugh-Nagumo equation among many others. By implementing the theory of C_0 -semigroups and evolution systems into the chaos expansion theory in infinite dimensional spaces, we prove existence and uniqueness of solutions for this class of SPDEs. Additionally, we treat the linear nonautonomous case and provide several applications featured as stochastic reaction-diffusion equations that arise in biology, medicine and physics.

The talk is based on collaborations with T. Levajković, S. Pilipović and D. Seleši.

Koneksije na nesimetričnim Rimanovim mnogostrukostima

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U radu predstavljamo koneksije sa totalno anti-simetričnom torzijom na nesimetričnoj Rimanovoj mnogostrukosti koje zadovoljavaju Ajnštajnovu metričku jednačinu

$$\frac{\partial G_{ij}}{\partial x^m} - \Gamma_{im}^p G_{pj} - \Gamma_{mj}^p G_{ip} = 0, \tag{1}$$

gde je Γ_{jk}^i koneksija a G_{ij} osnovni metrički tenzor. Pokazaćemo da je skoro Hermitska mnogostrukost koja zadovoljava Ajnštajnovu metričku jednačinu blizu Kelerova mnogostrukost i obratno. Bavićemo se skoro kontaktnim metričkim mnogostrukostima koje zadovoljavaju Ajnštajnovu metričku jednačinu. Većina dobijenih rezultata je u funkciji odgovarajućih Nijenhuis tenzora i spoljnog proizvoda anti-simetričnog dela nesimetrične Rimanove metrike. Pokazaćemo da koneksija sa totalno anti-simetričnom torzijom na nesimetričnoj Rimanovoj mnogostrukosti koje zadovoljavaju Ajnštajnovu metričku jednačinu postoji na skoro kontaktnoj metričkoj mnogostrukosti ako je ona D-homotetična kosimplektnoj mnogostukosti. Dalje istraživanje ćemo sprovesti na mnogostrukosti dimenzije 5.

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Let M be a square matrix of order n . The resolvent matrix $\mathcal{R}_M(z)$, of matrix M is defined as $\mathcal{R}_M(z) = (zI_n - M)^{-1}$, where I_n is the unit matrix of order n and z is a complex variable. Let G be a simple graph, and let L be its Laplacian matrix. Eigenvalues of matrix L we denote by $\mu_1 \geq \mu_2 \geq \dots \geq \mu_n$. We consider resolvent matrix $\mathcal{R}_L(n+1)$. The Laplacian resolvent energy of a graph G is defined as $RL(G) = \sum_{i=1}^n \frac{1}{n+1-\mu_i}$. In this paper we consider some properties of the invariant $RL(G)$ such as bounds and the extremal graphs.